Calcula la derivada de las siguientes funciones:

a) $y = \dfrac{x^2 - 3}{x^2 + 3}$

b) $y = \dfrac{x + 1}{(2 - x)^2}$

c) $y = \dfrac{3x^2}{x + \sqrt{x}}$

d) $y = \left(0{,}5 - \dfrac{x}{10}\right)^4$

a) $y' = \dfrac{2x \cdot (x^2 + 3) - (x^2 - 3) \cdot 2x}{(x^2 + 3)^2} = \dfrac{12x}{(x^2 + 3)^2}$

b) $y' = \dfrac{(2 - x)^2 + (x + 1) \cdot 2(2 - x)}{(2 - x)^4} = \dfrac{x + 4}{(2 - x)^3}$

c) $y' = \dfrac{6x \cdot (x + \sqrt{x}) - 3x^2 \cdot \left(1 - \dfrac{1}{2\sqrt{x}}\right)}{(x + \sqrt{x})^2} = \dfrac{15x^2 + 6x^2 \cdot \sqrt{x}}{2\sqrt{x} \cdot (x + \sqrt{x})^2}$

d) $y' = \dfrac{-4}{10} \cdot \left(0{,}5 - \dfrac{x}{10}\right)^3 = \dfrac{-2}{5} \cdot \left(0{,}5 - \dfrac{x}{10}\right)^3$

Halla la derivada de estas funciones:

a) $y = \dfrac{x^3}{(x + 1)^2}$

b) $y = \left(\dfrac{x^2 + 1}{x}\right)^3$

c) $y = \dfrac{x}{(2x + 1)^3}$

d) $y = \dfrac{1 - x^2}{x^2 - 4x + 4}$

a) $y' = \dfrac{3x^2 \cdot (x+1)^2 - x^3 \cdot 2 \cdot (x+1)}{(x+1)^4} = \dfrac{x^2 \cdot (x+3)}{(x+1)^3}$

b) $y' = 3 \cdot \left(\dfrac{x^2+1}{x}\right)^2 \cdot \dfrac{2x \cdot x - (x^2+1)}{x^2} = 3 \cdot \left(\dfrac{x^2+1}{x}\right)^2 \cdot \dfrac{x^2 - 1}{x^2}$

c) $y' = \dfrac{(2x+1)^3 - x \cdot 3(2x+1)^2}{(2x+1)^6} = \dfrac{(2x+1) - 6x}{(2x+1)^4} = \dfrac{1 - 4x}{(2x+1)^4}$

d) $y' = \dfrac{-2x(x^2 - 4x + 4) - (1 - x^2)(2x - 4)}{(x^2 - 4x + 4)^2} = \dfrac{-2x(x-2)^2 - (1 - x^2) \cdot 2(x-2)}{(x-2)^4} =$

$= \dfrac{-2x(x-2) - (1 - x^2) \cdot 2}{(x-2)^3} = \dfrac{4x - 2}{(x-2)^3}$

Deriva las funciones siguientes:

a) $y = e^{4x}(x - 1)$

b) $y = \dfrac{(1 - x)^2}{e^x}$

c) $y = \sqrt{2^x}$

d) $y = ln\,(2x - 1)$

a) $y' = 4 \cdot e^{4x} \cdot (x - 1) + e^{4x} \cdot 1 = e^{4x} \cdot (4x - 3)$

b) $y' = \dfrac{-2 \cdot (1 - x) \cdot e^x - (1 - x)^2 \cdot e^x}{e^{2x}} = \dfrac{-2 \cdot (1 - x) - (1 - x)^2}{e^x} = \dfrac{-x^2 + 4x - 3}{e^x}$

c) $y' = \dfrac{2^x \cdot ln2}{2\sqrt{2^x}} = \dfrac{2^{x-1} \cdot ln2}{\sqrt{2^x}}$

d) $y' = \dfrac{2}{2x - 1}$

Deriva estas funciones:

a) $y = ln\,(x^2 - 1)$ **b)** $y = ln\,\sqrt{1 - x}$ **c)** $y = \dfrac{ln\,x}{e^x}$ **d)** $y = e^{x^2 + 1}$

a) $y' = \dfrac{2x}{x^2 - 1}$

b) $y' = \dfrac{\dfrac{-1}{2\sqrt{1 - x}}}{\sqrt{1 - x}} = \dfrac{-1}{2(1 - x)}$

c) $y' = \dfrac{\dfrac{1}{x} \cdot e^x - ln\,x \cdot e^x}{e^{2x}} = \dfrac{\dfrac{1}{x} - ln\,x}{e^x} = \dfrac{1 - x \cdot ln\,x}{x \cdot e^x}$

d) $y' = 2x\,e^{x^2 + 1}$

Deriva las funciones siguientes:

a) $y = log_2 \dfrac{1}{x}$ **b)** $y = \sqrt[3]{sen\,x^2}$ **c)** $y = \sqrt{\dfrac{1 + 2x}{1 - 2x}}$ **d)** $y = \sqrt{x + \sqrt{x}}$

a) $y = log_2\,1 - log_2\,x$

$y' = -\dfrac{1}{x} \cdot \dfrac{1}{ln\,2} = \dfrac{-1}{x\,ln\,2}$

b) $y' = \dfrac{2x \cdot cos\,x^2}{3\sqrt[3]{sen^2\,x^2}}$

c) $y' = \dfrac{\dfrac{2 \cdot (1 - 2x) + (1 + 2x) \cdot 2}{(1 - 2x)^2}}{2 \cdot \sqrt{\dfrac{1 + 2x}{1 - 2x}}} = \dfrac{\dfrac{4}{(1 - 2x)^2}}{2 \cdot \sqrt{\dfrac{1 + 2x}{1 - 2x}}} = \dfrac{2}{(1 - 2x)^2 \cdot \sqrt{\dfrac{1 + 2x}{1 - 2x}}} =$

$= \dfrac{2}{\sqrt{(1 - 2x)^4 \cdot \dfrac{1 + 2x}{1 - 2x}}} = \dfrac{2}{\sqrt{(1 - 2x)^3 (1 + 2x)}}$

d) $y' = \dfrac{1 + \dfrac{1}{2\sqrt{x}}}{2 \cdot \sqrt{x + \sqrt{x}}} = \dfrac{2\sqrt{x} + 1}{4\sqrt{x} \cdot \sqrt{x + \sqrt{x}}} = \dfrac{2\sqrt{x} + 1}{4 \cdot \sqrt{x^2 + x\sqrt{x}}}$

Halla la derivada de:

a) $y = \sqrt{x\sqrt{x}}$

b) $y = ln\sqrt{\dfrac{x}{x + 1}}$

c) $y = ln\,(sen\,\sqrt{e^x})$

d) $y = \sqrt{\dfrac{x - 1}{x + 1}}$

a) $y = \sqrt{\sqrt{x^2 \cdot x}} = \sqrt[4]{x^3} = x^{3/4} \rightarrow y' = \dfrac{3}{4} \cdot x^{-1/4}\,\dfrac{3}{4 \cdot \sqrt[4]{x}}$

b) $y = \dfrac{1}{2} \cdot (ln\,x - ln\,(x + 1))$

$y' = \dfrac{1}{2} \cdot \left(\dfrac{1}{x} - \dfrac{1}{x + 1}\right) = \dfrac{1}{2x^2 + 2x}$

c) $y = ln\,(sen\,e^{x/2}) \rightarrow y' = \dfrac{(1/2) \cdot e^{x/2} \cdot cos\,e^{x/2}}{sen\,e^{x/2}} = \dfrac{e^{x/2} \cdot cos\,\sqrt{e^x}}{2 \cdot sen\,\sqrt{e^x}}$

d) $y' = \dfrac{\dfrac{x + 1 - x + 1}{(x + 1)^2}}{2 \cdot \sqrt{\dfrac{x - 1}{x + 1}}} = \dfrac{1}{(x + 1)^2 \cdot \sqrt{\dfrac{x - 1}{x + 1}}} = \dfrac{1}{\sqrt{(x + 1)^4 \cdot \dfrac{x - 1}{x + 1}}} =$

$= \dfrac{1}{\sqrt{(x - 1) \cdot (x + 1)^3}}$