Calcula la derivada de las funciones siguientes:

a) $f(x)=\operatorname{arcsen}\left(e^{x}\right)$

b) $f(x)=\operatorname{arctg}\left(1+x^{2}\right)$

c) $f(x)=\sqrt{\arccos x}$

d) $f(x)=\ln(\operatorname{sen}x+\operatorname{arcsen}x)$

a) $f'(x)=\dfrac{e^{x}}{\sqrt{1-e^{2x}}}$

b) $f'(x)=\dfrac{2x}{\left(1+x^{2}\right)^{2}}$

c) $f'(x)=-\dfrac{1}{2\sqrt{\arccos x}}\cdot\dfrac{1}{\sqrt{1-x^{2}}}$

d) $f'(x)=\dfrac{1}{\operatorname{sen}x+\operatorname{arcsen}x}\cdot\left(\cos x+\dfrac{1}{\sqrt{1-x^{2}}}\right)$

(TIC) Calcula la derivada de las siguientes funciones compuestas:

a) $f(x)=\left(\sqrt{x}+x\right)^{5}$

b) $f(x)=\sqrt{\dfrac{2x+1}{x^{3}}}$

c) $f(x)=\left(\dfrac{x+3}{x-5}\right)^{3}$

d) $f(x)=\dfrac{x^{2}}{(3x-6)^{4}}$

a) $f'(x)=5\left(\sqrt{x}+x\right)^{4}\left(\dfrac{1}{2\sqrt{x}}+1\right)$

b) $f'(x)=\dfrac{1}{2}\sqrt{\dfrac{x^{3}}{2x+1}}\dfrac{-4x-3}{x^{4}}$

c) $f'(x)=3\left(\dfrac{x+3}{x-5}\right)^{2}\dfrac{-8}{(x-5)^{2}}=-\dfrac{8}{3}\dfrac{(x+3)^{2}}{(x-5)^{4}}$

d) $f'(x)=\dfrac{2x(3x-6)-12x^{2}}{(3x-6)^{5}}=\dfrac{-6x(x+2)}{(3x-6)^{5}}$

(TIC) Calcula las derivadas de estas funciones:

a) $f(x)=3+x^{2}$

b) $f(x)=\dfrac{3}{x^{2}}$

c) $f(x)=\ln\left(\dfrac{3+x^{2}}{3}\right)$

d) $f(x)=e^{3+x^{2}}$

e) $f(x)=\left(3-x^{2}\right)^{3}$

f) $f(x)=\operatorname{sen}^{3}\left(x^{2}\right)$

g) $f(x)=3\operatorname{arcsen}\left(\dfrac{1}{x^{2}}\right)$

h) $f(x)=\left(\operatorname{sen}(x^{2})+\cos 3\right)^{3}$

a) $f'(x)=2x$

b) $f'(x)=\dfrac{-6}{x^{3}}$

c) $f'(x)=\dfrac{2x}{3+x^{2}}$

d) $f'(x)=2xe^{3+x^{2}}$

e) $f'(x)=-6x\left(3-x^{2}\right)^{2}$

f) $f'(x)=6x\operatorname{sen}^{2}\left(x^{2}\right)\cos(x^{2})$

g) $f'(x)=\dfrac{-3}{x\sqrt{x^{4}-1}}$

h) $f'(x)=3\left(\operatorname{sen}(x^{2})+\cos 3\right)^{2}\left(2x\cos(x^{2})\right)$

$$f(x)=\sqrt[3]{x+3}+\frac{1}{\sqrt[3]{x+2}}$$

f) $f(x)=\dfrac{\text{sen}^2(2x)+2\text{sen}x\cos x+\cos^2(2x)}{\text{sen}(2x)}$

$$f'(x)=\frac{\cos x\cdot\text{arctg}\,x-\dfrac{\text{sen}\,x}{1+x^2}}{(\text{arctg}\,x)^2}$$

$$f'(x)=e^x(\cos x-\text{sen}x)+e^{-x}(-\text{sen}x+\cos x)=(\cos x-\text{sen}x)(e^x+e^{-x})$$

a) $f'(x)=\dfrac{\cos x\cdot\text{arctg}\,x-\dfrac{\text{sen}\,x}{1+x^2}}{(\text{arctg}\,x)^2}$

b) $f'(x)=e^x(\cos x-\text{sen}x)+e^{-x}(-\text{sen}x+\cos x)=(\cos x-\text{sen}x)(e^x+e^{-x})$

c) $f'(x)=\dfrac{1}{3\left(\sqrt[3]{x+2}\right)^2}-\dfrac{1}{3\left(\sqrt[3]{x+2}\right)^4}$

d) $f'(x)=\dfrac{(2x\ln x+x)(x^2-1)-2x^3\ln x}{(x^2-1)^2}$

e) $f'(x)=\cos\left(\left(\text{sen}^3x^3+1\right)^3\right)3\left(\text{sen}^3x^3+1\right)^2 3\text{sen}^2x^3\cos x^3\cdot 3x^2=$

$$=27\cos\left(\left(\text{sen}^3\left(x^3\right)+1\right)^3\right)\left(\text{sen}^3\left(x^3\right)+1\right)^2\text{sen}^2\left(x^3\right)\cos\left(x^3\right)x^2$$

f) Antes de derivar conviene escribir la función de forma más sencilla utilizando las propiedades de las razones trigonométricas:

$$f(x)=\frac{\text{sen}^2(2x)+2\text{sen}\,x\cos x+\cos^2(2x)}{\text{sen}(2x)}=\frac{1+2\text{sen}\,x\cos x}{\text{sen}(2x)}=\frac{1+\text{sen}(2x)}{\text{sen}(2x)}=\frac{1}{\text{sen}(2x)}+1\Rightarrow f'(x)=\frac{-2\cos(2x)}{(\text{sen}(2x))^2}$$

(TIC) Halla las derivadas de las siguientes funciones:

a) $f(x)=\ln(x^2+1)\cdot\text{tg}\,x$

b) $f(x)=\text{sen}\left(\sqrt{x^2+4}\right)$

c) $f(x)=\dfrac{2\pi^2\ln\left(x^3+x\right)}{\text{tg}(\pi x)}$

d) $f(x)=\dfrac{e^{x2}}{\sqrt{x-1}}$

e) $f(x)=\dfrac{\text{arcsen}\,(3^x)}{x\cos^3\left(e^x\right)}$

f) $f(t)=\dfrac{\sqrt{te^t}}{\sqrt[4]{t^3-t^2}}$

a) $f'(x)=\dfrac{2x}{x^2+1}\cdot\text{tg}\,x+\dfrac{\ln(x^2+1)}{\cos^2 x}$

b) $f'(x)=\dfrac{x\cos\left(\sqrt{x^2+4}\right)}{\sqrt{x^2+4}}$

c) $f'(x)=\dfrac{\dfrac{2\pi^2\left(3x^2+1\right)\text{tg}(\pi x)}{x^3+x}-\dfrac{2\pi^3\ln\left(x^3+x\right)}{\cos^2(\pi x)}}{(\text{tg}(\pi x))^2}$

d) $f'(x)=\dfrac{e^{x^2}\left(2x\sqrt{x-1}-\dfrac{1}{2\sqrt{x-1}}\right)}{x-1}=\dfrac{e^{x^2}(4x(x-1)-1)}{2(x-1)\sqrt{x-1}}$

e) $f'(x)=\dfrac{\dfrac{\ln 3\cdot\left(x\cos^3\left(e^x\right)\right)3^x}{\sqrt{1-3^{2x}}}-\text{arcsen}\,(3^x)\left(\cos^3\left(e^x\right)-3xe^x\cos^2(e^x)\text{sen}(e^x)\right)}{\left(x\cos^3\left(e^x\right)\right)^2}$

f) $f'(t)=\dfrac{\dfrac{e^t(1+t)\sqrt[4]{t^3-t^2}}{2\sqrt{te^t}}-\dfrac{\sqrt{te^t}\left(3t^2-2t\right)}{\left(\sqrt[4]{t^3-t^2}\right)^3}}{\sqrt{t^3-t^2}}$

1. $f(x)=(1+3x^4)^5 \Rightarrow f'(x)=60\,x^3(1+3x^4)^4$
2. $f(x)=(1+x+x^2)^3 \Rightarrow f'(x)=3(2x+1)(1+x+x^2)^2$
3. $f(x)=\dfrac{1}{(x^2-1)^4} \Rightarrow f'(x)=-8x(x^2-1)^{-5}$
4. $f(x)=\dfrac{1}{x-1}+\dfrac{2}{(x-1)^2}+\dfrac{3}{(x-1)^3} \Rightarrow f'(x)=-(x-1)^{-2}-4(x-1)^{-3}-9(x-1)^{-4}$
5. $f(x)=\sqrt{1-x^2} \Rightarrow f'(x)=\dfrac{-x}{\sqrt{1-x^2}}$
6. $f(x)=\sqrt[3]{2+5x^2} \Rightarrow f'(x)=\dfrac{10x}{3\sqrt[3]{(2+5x^2)^2}}$
7. $f(x)=\dfrac{1}{\sqrt[3]{(x^3-2)^2}} \Rightarrow f'(x)=-2x^2(x^3-2)^{-5/3}$
8. $f(x)=(5x^3+1)^3\cdot(x^2+x+1)^4 \Rightarrow$
   $f'(x)=45x^2(5x^3+1)^2(x^2+x+1)^4+4(2x+1)(5x^3+1)^3(x^2+x+1)^3$
9. $f(x)=(5-3\cos x)^4 \Rightarrow f'(x)=12\,\mathrm{sen}\,x(5-3\cos x)^3$
10. $f(x)=\mathrm{sen}\,x+\mathrm{sen}^2x+\mathrm{sen}^3x \Rightarrow f'(x)=\cos x(1+2\,\mathrm{sen}\,x+3\,\mathrm{sen}^2x)$
11. $f(x)=\dfrac{1}{\mathrm{arc\,tg}\,x} \Rightarrow f'(x)=\dfrac{-1}{(1+x^2)(\mathrm{arc\,tg}\,x)^2}$
12. $f(x)=\mathrm{sen}^3x-\cos^3x \Rightarrow f'(x)=3\,\mathrm{sen}x\cos x(\mathrm{sen}x+\cos x)$
13. $f(x)=\dfrac{1}{3\cos^3x}-\dfrac{1}{\cos x} \Rightarrow f'(x)=\mathrm{sen}x\left[\dfrac{1}{\cos^4x}-\dfrac{1}{\cos^2x}\right]$
14. $f(x)=\mathrm{sen}(x^2) \Rightarrow f'(x)=2x\cos(x^2)$
15. $f(x)=(1+\mathrm{sen}\,5x)^4 \Rightarrow f'(x)=20\cos 5x(1+\mathrm{sen}5x)^3$
16. $f(x)=\sqrt{x\,e^x+x} \Rightarrow f'(x)=\dfrac{e^x+xe^x+1}{2\sqrt{xe^x+x}}$
17. $f(x)=\sqrt[3]{2^x+x} \Rightarrow f'(x)=\dfrac{2^x\mathrm{Ln}2+1}{3\sqrt[3]{(2^x+x)^2}}$
18. $f(x)=\mathrm{Ln}(\mathrm{Ln}\,x) \Rightarrow f'(x)=\dfrac{1}{x\,\mathrm{Ln}x}$
19. $f(x)=\mathrm{Arc}\cos\sqrt{x} \Rightarrow f'(x)=\dfrac{-1}{2\sqrt{x-x^2}}$
20. $f(x)=\mathrm{Arc\,sen}\left(\dfrac{1}{x^2}\right) \Rightarrow f'(x)=\dfrac{-2}{x\sqrt{x^4-1}}$
21. $f(x)=\dfrac{1+\cos 2x}{1-\cos 2x} \Rightarrow f'(x)=\dfrac{-4\,\mathrm{sen}\,2x}{(1-\cos 2x)^2}$
22. $f(x)=\mathrm{Arc\,tg}\left(\dfrac{1}{x}\right) \Rightarrow f'(x)=\dfrac{-1}{1+x^2}$
23. $f(x)=\mathrm{Arc\,tg}(e^x) \Rightarrow f'(x)=\dfrac{e^x}{1+e^{2x}}$
24. $f(x)=\mathrm{Arc\,sen}\left(\dfrac{x^2-1}{x^2}\right) \Rightarrow f'(x)=\dfrac{2}{x\sqrt{2x^2-1}}$
25. $f(x)=\mathrm{Ln}\,(\mathrm{sen}\,x) \Rightarrow f'(x)=\cot x$
26. $f(x)=\mathrm{sen}^3x\cdot\cos^3x \Rightarrow f'(x)=3\,\mathrm{sen}^2x\cos^2x(\cos^2x-\mathrm{sen}^2x)$